Understanding Heat Transfer Coefficient

When there is a motion of fluid with respect to a surface or a gas with heat generation, the transport of heat is referred to as convection [1]. There are three modes of convection. If the motion of flow is generated by external forces, such as a pump or fan, it is referred to as forced convection. If it is driven by gravity forces due to temperature gradients, it is called natural (or free) convection. When the external means are not strong and gravitational forces are strong, the resulting convection heat transfer is called mixed convection.

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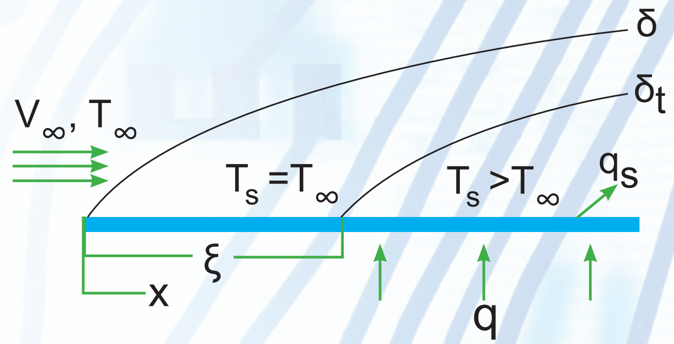
A heat transfer coefficient h is generally defined as:

$$Q = hA(T_s - T_\infty)$$

Where **Q** is the total heat transfer, **A** is the heated surface area, T_s is the surface temperature and T_∞ is the approach fluid temperature. For the convection heat transfer to have a physical meaning, there must be a temperature difference between the heated surface and the moving fluid. This phenomenon is referred to as the thermal boundary layer that causes heat transfer from the surface. In addition to the thermal boundary layer, there is also a velocity boundary layer due to the friction between the surface and the fluid induced as the result of the fluid viscosity. The combination of the thermal and viscous boundary layers governs the heat transfer from the surface. Figure 1 shows velocity boundary layer growth (δ) that starts from the leading edge of the plate. The thermal boundary layer (δ_t) starts after a distance (ξ) from where the temperature of the plate changes from ambient temperature to a different temperature (T_s) , causing convection heat transfer.

Figure 1.
Velocity and thermal boundary layer growth on a heated flat plate [4].



To relate these two parameters, an important equation known as the Reynolds Analogy can be derived from conservation laws that relate the heat transfer coefficient to the friction coefficient, C_f:

$$C_f \frac{\mathbf{Re}_L}{\mathbf{2}} = Nu$$

Where Nu is the Nusselt number, Re_L is the Reynolds number based on a length scale of L and C_f is defined as:

$$\mathbf{C_f} = \frac{\tau_s}{\rho \mathbf{V}^2/\mathbf{2}}$$

Where τ_s is the shear stress and **V** is the reference velocity, the shear stress τ_s can be calculated as:

$$\tau_s = \mu \frac{dV}{dy}\bigg|_{y=0}$$

Where μ is the fluid viscosity, and the derivative of the velocity (V) is calculated at the wall. The above equation is significant because it allows the engineer to obtain information about the heat transfer coefficient by knowing the surface friction coefficient and vice versa. To calculate the friction coefficient, the velocity gradient at the wall is needed. In a simple flat plate, the definition of the heat transfer coefficient is well defined, if we assume the plate has a constant temperature and the fluid temperature outside the boundary layer is fixed.

From the definition above, h purely depends on the fluid and surface reference temperatures. In simple geometries, h can be analytically derived based on the conservation equations. For example, for the laminar flow over a flat plate, analytical derivation for the local heat transfer coefficient yields

$$h_x = \frac{0.332}{x} k_f \, Re_x^{1/2} Pr^{1/3}$$

for Prandtle numbers above 0.6. The Prandtl number, **Pr**, is the ratio of momentum diffusivity (viscosity) to thermal diffusivity. The **Pr** for liquid metals is much less than 1, for gases it is close to 1, and for oils it is much higher than 1. The Pr for air at atmospheric pressure and 27°C is 0.707.

For a fully developed laminar flow in a circular tube, the analytical form for the Nusselt number is:

$$Nu \equiv \frac{hD}{k} = 4.36$$

for a constant temperature surface, and

$$Nu = 3.66$$

for a constant heat flux.

The above values for a circular duct are not valid in the entry region of the tube, where the velocity or thermal boundary layer is still developing (ξ length in figure 1). In the entry region, the heat transfer coefficient is much higher than in the fully developed region. Figure 2 shows the Nusselt number Nu, as a function of the inverse of the Graetz number.

The Graetz number is defined as:

$$Gz_D = \left(\frac{D}{x}\right) Re_D \, Pr$$

Where x is the distance from the leading edge, and Re_D is the Reynolds number based on the duct diameter. This figures shows the Nu in an entry length region where the velocity is already fully developed, and the combined entry length which both the velocity and the temperature are developing. It shows that if the flow and the temperature are developing at the same time, the Nusselt number would have been higher than the thermal entry length.

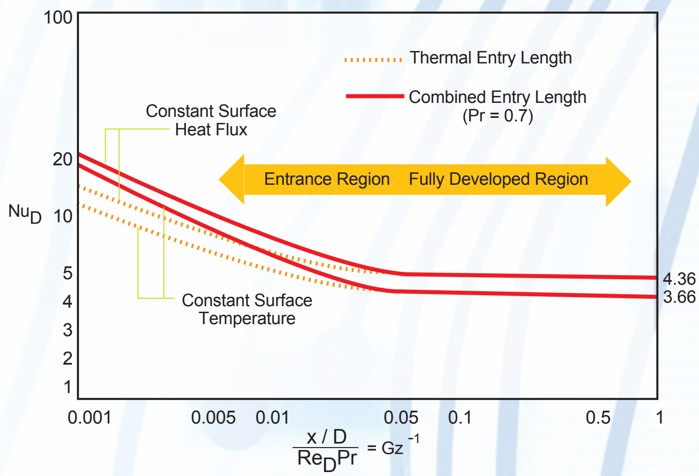


Figure 2.
The Nusselt number as a function of the inverse of the Graetz number in a duct [1].

The heat transfer coefficient also depends on the flow regime. Figure 3 shows the flow over a flat surface. The laminar boundary layer starts the transition at a Reynolds number around 5 X 10^5 with a sudden jump in the heat transfer coefficient, and then gradually coming down in the turbulent region, but still above the laminar regime. For a smooth circular tube, the transition from laminar to turbulent starts at a Reynolds number around 2300.

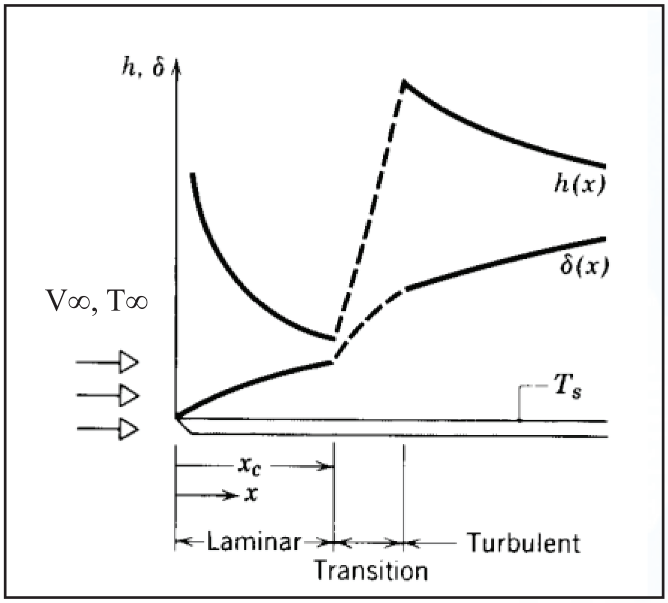


Figure 3.
Heat transfer coefficient on a flat plate for different flow regimes [4].

An important issue is the definition of the ambient and fluid temperatures. The user must be careful when using heat transfer coefficient correlations by knowing where the reference ambient temperature is defined. For example, in [2] Azar and Moffat (figure 4.) considered the flow between circuit boards in an electronic enclosure as the reference ambient fluid temperature. In this case, there are many choices for defining the heat transfer coefficient, depending on how the reference temperatures are defined. The surface temperature is more fully defined, as it can be assumed to be either the hottest point on the chip or some area average of the surface temperature. The fluid temperature, however, is harder to define because there are many choices. The authors assumed three choices for the fluid temperature:

1- T_{in} as the inlet fluid temperature

2- T_m based on the inlet temperature and upstream heat dissipation

$$T_m = T_{in} + \sum \dot{q} / \dot{m}\ C_p$$

Where $\dot{m}$ is the mass flow rate and $\dot{q}$ is the upstream heat dissipation.

3- T_{ad} (adiabatic temperature) as the gas temperature measured by a device with no power.

In their experiment, they used an array of 3 x 3 components and measured the heat transfer coefficients for different powering schemes. The results revealed that the h_{in} and h_m based on T_{in} and T_m resulted in 25% variation between the mean value, but the variation of h_{ad} based on T_{ad} was about 5% around the mean. The authors concluded that h_{in} and h_m depend on the other component powering schemes and the temperature distribution of the system. But, h_{ad} depends on the aerodynamics of the flow near the component and is independent of the powering scheme.

This same argument applies to a heat sink. One has to be careful how the heat transfer coefficient is defined. It can be defined in reference to inlet fluid temperature, or as some mean value between the inlet and outlet. If the heat transfer coefficient is defined based on the inlet temperature, the thermal resistance of the heat sink is calculated as:

$$R = \frac{1}{\eta h A}$$

Where A is the total surface area, and η is the fin efficiency defined as:

$$\eta = \frac{\tanh(m \times H)}{m \times H} \quad \text{and} \quad m = \sqrt{\frac{h \times P}{K \times A}}$$

Where H is fin height, h is the heat transfer coefficient, P is the fin perimeter, A is the fin cross sectional area and K is the fin thermal conductivity.

But, if the heat transfer coefficient is defined based on the temperature in between the fins, the thermal resistance calculation also involves the addition of the heat capacitance of the flow as:

$$R = \frac{1}{\eta h A} + \frac{1}{2 \dot{m} C_p}$$

Where C_p is the fluid heat capacitance.

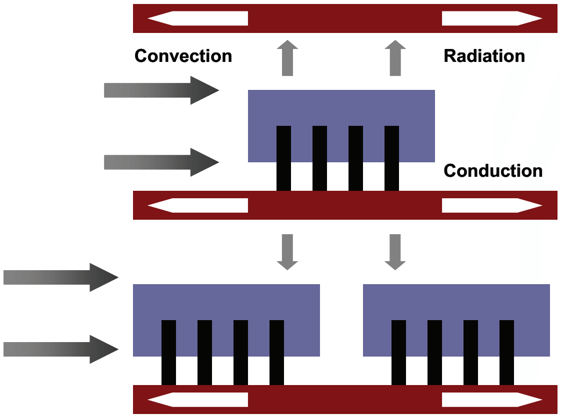


Figure 4. Components placed on a PCB inside a Chassis [2].

The magnitude of the heat transfer coefficient is impacted by a number of parameters such as geometry, flow rate, flow condition, and fluid type. Figure 5 shows the heat transfer coefficients for some common liquids and the modes of heat transfer. It shows an approximate heat transfer coefficient of 10 W/m^{2o}C for air in natural convection to around 100,000 W/m^{2o}C for water in pool boiling mode. Perhaps the best transport condition, about 200,00 W/m^{2o}C, is seen in the water condensation mode. Recent advances in microchannels show that a heat transfer coefficient of close to 500,000 W/m^{2o}C can be achieved.

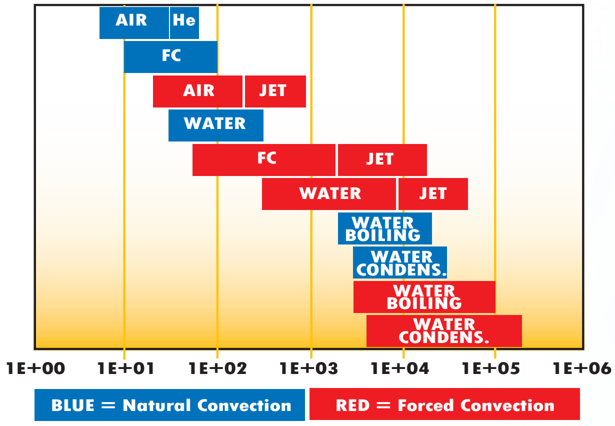


Figure 5. Heat transfer coefficients for some common liquids and different modes [3].

In summary, before using any heat transfer correlation from the literature, the engineer must determine the flow conditions and use the appropriate equations. These flow conditions can be categorized as: 1- Laminar or turbulent, 2- Entry length, fully developed or both, 3- Internal or external flow, 4- Natural convection, forced convection, jet impingement, boiling, spray, etc.

For complicated geometries and different flow regimes, there are a vast number of empirical equations that have been published by different researchers. The reader is cautioned to use these correlations carefully since they are all defined for a specific range of parameters. This includes such numbers as Reynolds, Prandtle, Peclet in the case of liquid metals, the ratio of some parameters such as length to diameter and the Rayleigh number, in case of natural convection.

References:

1. Kays, W.M., Convective heat and mass transfer, 1966.
2. Azar, K, and Moffat, R., Evaluation of different heat transfer coefficients definitions, Electronics Cooling, June 1995, Volume 1.1, Number 1.
3. Lasance, C. and Moffat, C., Advances in high-performance cooling for electronics Electronics Cooling, November 2005, Volume 11, Number 4.
4. Incropera, F.P. and Dewitt, D.P., Introduction to Heat Transfer, 1985.

Nomenclature:

Q	Total heat transfer (W)
h	Heat transfer coefficient (W/m^{2o}C)
T_s	Surface temperature (^{o}C)
T_∞	Ambient temperature (^{o}C)
C_f	Friction Coefficient
Re_L	Reynolds number based on reference length L
Re_D	Reynolds number based on the duct diameter
Nu	Nusselt number
t_s	Shear stress (N/m^2)
Pr	Prandtl number
K	Thermal conductivity(W/m^{o}C)
m	Mass flow rate (kg/sec)
R	Thermal resistance (^{o}C/W)
n	Fin efficiency
P	Fin perimeter (m)
A	Surface area (m^2)
A_f	Fin cross section area (m^2)
C_p	Thermal capacitance (^{o}C/W)
μ	Fluid dynamic viscosity (N.sec/m^2)
Gz_D	Graetz number based on duct diameter D